# Simulation of Flow and Electro-Magnetic Induced Vibrations

## - Mapping Procedures for NVH Analyses -

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## 1 Introduction

Structural vibrations are of special interest in a wide range of engineering applications. They produce noise in their surroundings or lead to a reduction of the product lifetime.
In order to predict these vibrations realistically using structural simulation techniques, a detailed knowledge of the excitation load is necessary. This can be achieved by a preceding simulation of the load-generating physical processes.
By a mapping procedure, these excitations are transferred to the structural mesh. As vibration analyses are performed in the frequency domain, complex scalars or vectors have to be supported.

## 2 Steady State Analyses

The linear equations of motion

$$M\ddot{x} + D\dot{x} + Kx = f$$

$M$ mass matrix
$D$ damping matrix
$K$ stiffness matrix

describe the transient displacement $x(t) \in \mathbb{R}^n$ of a $n$-degree of freedom system (e.g. finite element model) loaded by a force $f(t) \in \mathbb{R}^n$ [1].
In many problems the steady state solution of the equations is sought, i.e. without consideration of the initial phase of attack. The linearity of the equations allows to examine the responses to the single harmonic frequency components of the force. Superposing all the single responses leads to the response to the total force.
So, in order to formulate this mathematically, let $f$ be a harmonic force of frequency $\Omega$

$$f(t) = b \cdot \mathrm{e}^{\mathrm{i}(\Omega t+\vartheta)} = b\mathrm{e}^{\mathrm{i}\vartheta} \cdot \mathrm{e}^{\mathrm{i}(\Omega t)} = \tilde{f} \cdot \mathrm{e}^{\mathrm{i}(\Omega t)}$$

Due to the linearity, the structure responds in the same frequency

$$x(t) = a \cdot \mathrm{e}^{\mathrm{i}(\Omega t+\varphi)} = a\mathrm{e}^{\mathrm{i}\varphi} \cdot \mathrm{e}^{\mathrm{i}(\Omega t)} = \tilde{x} \cdot \mathrm{e}^{\mathrm{i}(\Omega t)}$$

Inserting both equations into the equations of motion leads to a linear complex system of equations

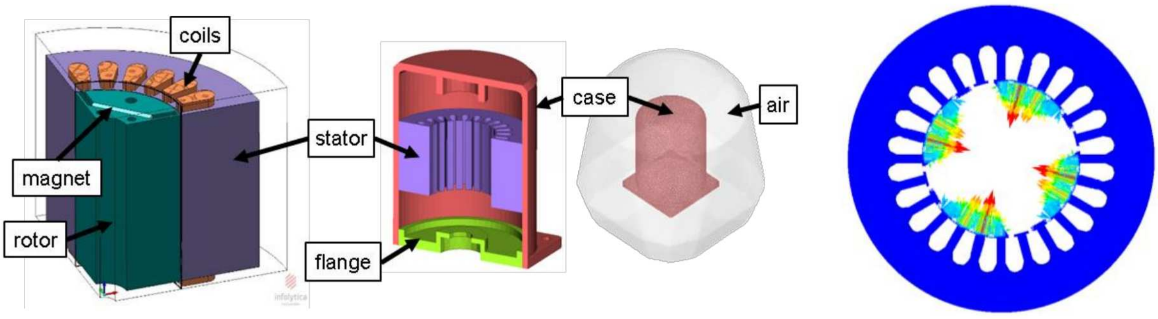
$$(-\Omega^2 M + \mathrm{i}\Omega D + K) \cdot \tilde{x} = \tilde{f}$$

where $\tilde{x}, \tilde{f} \in \mathbb{C}^n$ represent the complex amplitudes of the displacement resp. force oscillations. By Euler's formula, they can be reformulated as magnitude and phase lag of the oscillation at frequency $\Omega$. In this way, the equations of motion are transferred into the frequency domain. In steady state analyses, they are solved independently for each frequency component. The loading and the response is given in terms of complex frequency spectra.

## 3 Electro-Magnetic Induced Vibrations in E-Motors

In e-motors, the rotation of the electro-magnetic (EM) field induces periodically repeating forces which excite vibrations of the structure. The vibrations are transmitted to the motor housing and emit noise in the surrounding air.

A transient EM simulation with Infolytica's *MagNet* is performed for the 4-pole-24-slots electro motor shown in Fig. 1(a). The electro-magnetic forces evolving during one period (one quarter turn) are calculated, cf. Fig. 1(b).




(a) Quarter model of the electro motor in its casing surrouded by air

(b) Surface force density calculated by *MagNet* and extended the full model

Fig. 1: 4-pole-24-slots electro motor with simulated EM forces

As the resulting vibrations are simulated in *MSC.Nastran*, the following steps are performed in order to prepare the excitation data for a steady state analysis (see Fig. 2):

- map the transient data to the element discretization of the structural model
- transform the transient forces by Fourier into a complex force spectrum
- express the spectrum in the *MSC.Nastran* bulk data format

The process is implemented in *MpCCI FSIMapper* which can be realized by GUI or in batch.

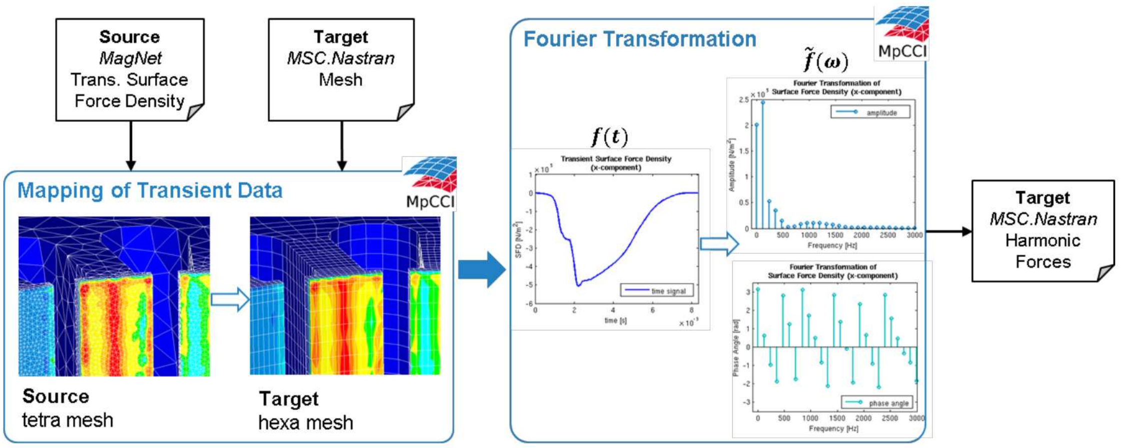



Fig. 2: Mapping process implemented in *MpCCI FSIMapper* including the Fourier transformation of the transient data

The generated excitation data (in terms of frequency dependent complex force amplitudes) are included into the *MSC.Nastran* simulation.

For each frequency component contained in the original transient signal, *MSC.Nastran* performs an independent calculation.

At the fourth harmonic of the rotation speed, the resulting displacement amplitude is the highest, cf. Fig. 3. This is explained by the four permanent magnets of the rotor.

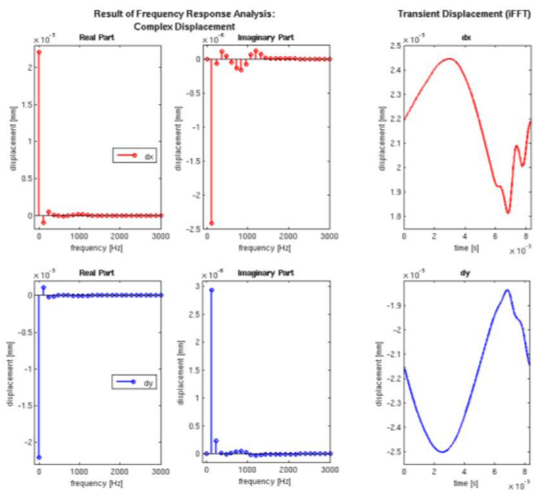



Fig. 3: Results of the steady state analysis

## 4 Flow Induced Vibrations in Turbomachinery

Due to interactions between rotating and stationary turbomachinery components, the transient fluid flow is characterized by periodic pressure fluctuations. Those oscillations excite vibrations of the blades which influence the high cycle fatigue (HCF) of the turbine component.
The periodic nature of the turbomachinery flow allows the approximation of the transient flow conditions by harmonic computational fluid dynamics (CFD) methods. In the Nonlinear Harmonic (NLH) method of *FINE/Turbo* the pressure oscillation $p(\vec{r},t)$ is represented by the time-averaged pressure $\bar{p}$ and a set of harmonic amplitudes $\tilde{p}_k$, each indicated by a certain harmonic frequency $\omega_k$ [2,3,4]:

$$p(\vec{r},t) = \bar{p}(\vec{r}) + \sum_{\substack{k=-K \\ k\neq 0}}^{K} \tilde{p}_k(\vec{r}) \cdot \mathrm{e}^{\mathrm{i}\omega_k t}$$

| | |
|---|---|
| $\vec{r}$ | position |
| $t$ | time |
| $p \in \mathbb{R}$ | pressure |
| $\bar{p} \in \mathbb{R}$ | time-averaged pressure |
| $\tilde{p}_k \in \mathbb{C}$ | $k$-th harmonic pressure amplitude |
| $\omega_k$ | corresponding $k$-th harmonic frequency |

The two-stage axial machine shown in Fig. 4 is simulated by the NLH method using $K = 16$ harmonics. The resulting complex pressure amplitudes (third harmonic for instance shown in Fig. 5) exhibit by definition the required form of the loading in steady state analyses.

The steady state simulation is performed in *ABAQUS* on a different mesh discretization and moreover on a different section shape and position, cf. Fig. 6.

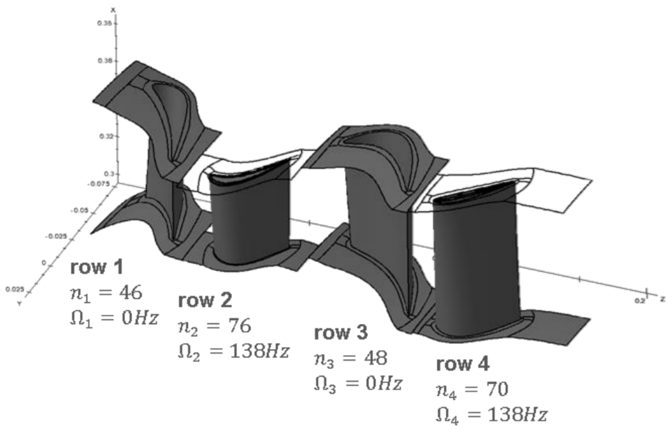



Fig. 4: Turbomachinery configuration

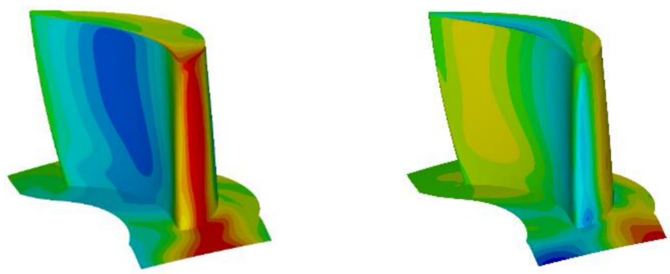
*MpCCI FSIMapper* uses the periodicity information of the data to map the set of harmonic pressure amplitudes $\{\tilde{p}_k \mid k = 1, \ldots, K\}$ from the *FINE/Turbo* result file to the *ABAQUS* mesh.
In general, dynamic excitations (as the harmonic pressure amplitudes) of cyclic symmetric systems cannot be described by a simple passage-to-passage periodicity. Moreover, “phase-shift” periodicities occur, which means, that in adjacent passages the dynamic amplitude is the same, but there is a constant temporal phase angle difference, the inter-blade phase angle $\sigma$.

The complex data $\tilde{p}^{(s)}$on sector $s$ is derived from sector $1$, by the following formula [5,6,7]:

$$\tilde{p}^{(s)} = \tilde{p}^{(1)} \cdot \mathrm{e}^{-\mathrm{i}(s-1)\sigma}$$

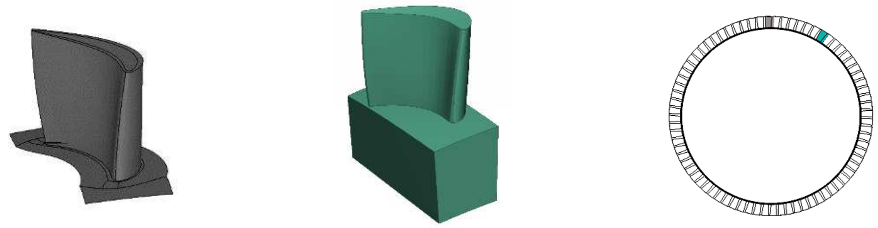
Which inter-blade phase angle is to be used for the pressure harmonic $\tilde{p}_k$, is determined by the influencing components and frequencies, which differ in general.

Exemplary, the resulting displacement and stress oscillation magnitudes of the third harmonic are shown in Fig. 7. The stress cycle amplitudes can be used in fatigue calculations.



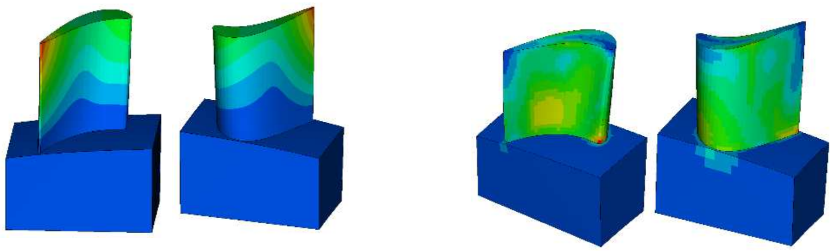
(a) real part (b) imaginary part

Fig. 5: Third complex pressure amplitude $\tilde{p}_3$ at 6350 Hz on row 2



(a) CFD section (b) FEM section (c) Comparison of CFD and FEM position in row 2

Fig. 6: Different section shapes and positions of the periodic models of row 2



(a) Displacement (b) Stress

Fig. 7: Displacement and stress magnitudes in row 2 loaded by the third pressure harmonic

## 5 Conclusions

The simulation data mapping tool *MpCCI FSIMapper* has been extended to support structural NVH analyses. It offers the Fourier transformation for transient data and the mapping of harmonic data. It enables the possibility to map between geometrically different periodic sections of a model under consideration of the data's phase-shift periodicity.

## 6 References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